Tutorial for Unit VI

1. Find Laplace transformation of the following functions:

   (i) t^n (ii) e^{at} (iii) $\cos at$ (iv) $\sin at$ (v) $\cosh at$ (vi) $\sinh at$

   (vii) $e^{at}\cos bt$ (viii) $e^{at}\sinh bt$ (ix) $t^2 e^{at}$ (x) $t^2 e^{2t}\cos 4t$

   (xi) $\cos^2 at$ (xii) $at^3 + bt^2 + ct + d$

2. Find Laplace transformation of

$$f(t) = \begin{cases} 1, & \text{if } 0 \le t < 2 \\ -1, & \text{if } 2 \le t < 4. \end{cases}$$

   where $f(t)$ is a periodic function with period 4, i.e. $f(t+4) = f(t)$.

3. Find inverse Laplace transformation of the following functions:

   (i) $\frac{1}{(s+a)^n}$ (ii) $\frac{s+2}{s^2-4s+13}$ (iii) $\frac{s^3}{(s^2+a^2)^2}$ (iv) $\ln\frac{s^2+1}{s(s+1)}$ (v) $\frac{s^2+2s+5}{(s-1)(s-2)(s-3)}$

4. Using Lapalce transformation solve the following IVP:

   (i) $y'' + 4y = 0$, $y(0) = 1$, $y'(0) = 6$
   (ii) $y'' + 2y' - 3y = 3$, $y(0) = 4$, $y'(0) = -7$
   (iii) $y'' - 5y' + 4y = e^{2t}$, $y(0) = \frac{19}{11}$, $y'(0) = \frac{8}{3}$
   (iv) $ty'' + y' + ty = 0$, $y(0) = 1$, $y'(0) = 0$

5. Find the Fourier series expansion of the periodic function defined by

$$f(x) = \begin{cases} 0, & \text{if } -\pi \le x < 0 \\ x, & \text{if } 0 \le x \le \pi. \end{cases}$$

6. Find the Fourier series expansion of the periodic function defined by

$$f(x) = |x| \quad \text{in} \quad -\pi \le x \le \pi.$$

7. Find the Fourier transforms of the following functions:

(i) $f(t) = \begin{cases} -(1+t), & -1 \le t \le 0 \\ t - 1, & 0 < t \le 1 \\ 0, & |t| > 1. \end{cases}$

(ii) $f(t) = \begin{cases} \cos t, & -l \le t \le l \\ 0, & \text{otherwise.} \end{cases}$

(iii) $f(t) = e^{-a|t+1|}$.

(iv) $f(t) = H(t-3)e^{-4t}$.

8. Find the inverse Fourier transforms of the following functions:

(i) $\dfrac{e^{-i\omega}}{2(1+i\omega)}$.

(ii) $\dfrac{e^{-2i\omega}}{4+\omega^2}$.

(iii) $\dfrac{1}{6+5i\omega-\omega^2}$.

9. Using convolution find the inverse Fourier transforms of the following functions:

(i) $\dfrac{1}{12+7i\omega-\omega^2}$.

(ii) $\dfrac{1}{(i\omega+k)^2}, k > 0$.

10. Using Fourier transforms solve the following differential equations:

(i) $y' - 4y = H(t)e^{-4t}, -\infty < t < \infty$.

(ii) $y' + 3y = H(t)e^{-2t}, -\infty < t < \infty$.